ON LIFT IN FREE MOLECULAR FLOW

(O POD'EMNOI SILE SVOBODNOMOLEKULIARNOM POTOKE)

PMM Vol.26, No.3, 1962, pp. 567

V.S. GALKIN
(Moscow)

(Received December 6, 1961)

In [1] it is shown that if the forces on the base of such bodies as wedges, cones, etc. are neglected, then the lift of these bodies in free molecular flow can be negative for all values of the angle of attack in the interval $0 < \alpha \leqslant \pi/2$ and of the criterion $\vartheta = V/c$, where V is the velocity of the body and c is the most probable thermal velocity of the oncoming molecules. In the present note it is shown that this conclusion is also valid for certain bodies of finite length taking into consideration the forces which are acting on their bases.

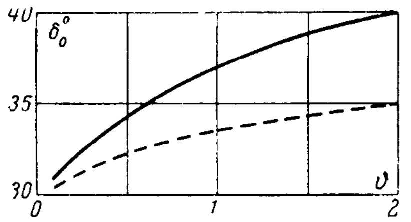
We shall consider as the simplest example the case of flow about a symmetrical wedge of finite length with semi-opening angle δ. As the basis of the conclusion about the negativeness of the lift it is necessary to demonstrate the existence of a value of $\delta = \delta_0$ such that for $\delta \geqslant \delta_0$ and $0 \leqslant \alpha \leqslant \pi/2$ the lift coefficient of the wedge $C_y \leqslant 0$ and for $\delta < \delta_0$ we have $C_y > 0$ in some interval of values of $\alpha > 0$, i.e. it is necessary to determine the value of $\delta = \delta_0$ from the condition $dC_y/d\alpha = 0$ for $\alpha = 0$.

Let the vector **V** lie in the plane of xy, which is perpendicular to the base of the wedge. The x-axis is antiparallel to **V**; the y-axis is obtained by rotating the x-axis counter-clockwise through a right angle; α is the angle between the x-axis and the plane of symmetry of the wedge; $(\pi/2 - \theta)$ is the angle between the outward normal to the body and the vector **V**. The lift Y is directed along the y-axis; q is the dynamic pressure; F is the base area of the wedge. The lift coefficient of the wedge is $C_y = Y/qF$. The formulas for the pressure p and the tangential stress τ acting on an element of the surface of the wedge have the form [2]

$$p = \frac{q}{2\vartheta^2}\left[\left(\sqrt{T} + \frac{2\eta}{\sqrt{\pi}}\right)\exp(-\eta^2) + \right.$$
$$\left. + (1 + \sqrt{\pi T}\,\eta + 2\eta^2)(1 + \operatorname{erf}\eta)\right] \qquad (1)$$

$$\tau = \frac{q\cos\theta}{\sqrt{\pi}\vartheta}\left[\exp(-\eta^2) + \sqrt{\pi}\eta(1+\operatorname{erf}\eta)\right]$$

$$\eta = \vartheta\sin\theta$$



Here T is the ratio of the temperature of the reflected molecules to the statistical temperature of the oncoming flow. Using (1) we find that the lift coefficient of an element of the surface, referred to the area of the element, and the lift coefficient of the wedge are respectively equal to

$$k = \frac{\cos\theta}{2\vartheta^2}\left[\sqrt{T}\exp(-\eta^2) + (1+\sqrt{\pi T}\eta)(1+\operatorname{erf}\eta)\right], \quad C_y = \frac{k_1+k_2}{2\sin\delta} + k_3 \qquad (2)$$

Here k_1, k_2, k_3 are respectively the lift coefficients of elements of the lower, upper and rear surfaces of the wedge. Hence we obtain the desired condition (for T = const)

$$\left(\sqrt{T} - \frac{2\vartheta}{\sqrt{\pi}}\frac{1-\sin^2\delta}{\sin\delta}\right)\exp(-\vartheta^2\sin^2\delta) - \sqrt{T}\exp(-\vartheta^2) +$$
$$+ [1+\operatorname{erf}(\vartheta\sin\delta)]\left(1 - \vartheta\sqrt{\pi T}\,\frac{1-2\sin^2\delta}{\sin\delta}\right) - (1-\operatorname{erf}\vartheta)(1-\vartheta\sqrt{\pi T}) = 0$$

The results of the solution of this equation are presented in the figure. The solid line indicates the data for $T = 1$, the dashed line for $T = 0.1$. For large values of ϑ we have the solution $\sin\delta_0 \approx 1/\sqrt{2} - (4\vartheta\sqrt{(\pi T)})^{-1}$, which with satisfactory accuracy is applicable for $\vartheta \geqslant 3$, $T \geqslant 1$. Thus, the magnitude of δ_0 essentially diminishes together with ϑ. In the general case the corresponding characteristic quantities analogous to δ_0 will naturally depend on the form of the body and the value of T.

BIBLIOGRAPHY

1. Galkin, V.S. and Gladkov, A.A., O pod'emnoi sile pri giperzvukovykh skorostiakh (On the lift force at hypersonic speeds). *PMM* Vol. 25, No. 6, 1961.

2. Hayes, W.D. and Probstein, R.F., *Hypersonic Flow Theory*. Academic Press, New York, 1959.

Translated by R.D.C.